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Applications Production Functions

Content

- Production Function
- Types of Production Functions
- Linear Production function
- Quadratic Production Function
- Cobb-Douglas Production Function
- Marginal value productivity (MVP)

Production Function

- The production function may be defined as mathematical relationship between input and output.

$$Y = f(X_1, X_2, \dots\dots\dots X_n)$$

- A systematic and mathematical way of measuring the relationship among different quantities of input or input services used in the production of commodity and corresponding quantities of output is called Production Function.
- **Use of production Function;**

✓To attain the equilibrium

✓To maximize the profit

✓To minimize the cost

✓Help in decision making etc.

Why study Production function?

- In estimating the quantity of output that may be expected when given input combined in specified manner.
- In planning production operation to locate the point of highest profit in farming.
- In computing physical input-output ratio to be used while preparing farm budget.
- In providing yard stuck of how efficient resource are being used on a farm under given conditions.

Criteria for Judging the function as best fit

- R^2 value (magnitude of the coefficient of multiple determination).
- T-test or F-test value, (as one may be the null hypothesis of regression coefficient at zero level).
- R^2 indicate the proportion of variation in the dependent variable accounted for by independent variable term which should be dropped nonsignificant at accepted probability level.
- A fitted function may be considered as the best-fit with a high value of R^2 and a relatively large number of significant constant coefficient.

Types of Production Functions

1. Linear Production function
2. Quadratic Production function
3. Cubic Production function
4. Constant Elasticity of Substitution Production Function
5. The power production function (Cobb-Douglas Production function)
6. The Spillman Production Function
7. The transcendental Production Function

Linear Production function

- The Linear Production function is the simplest type of production function. It can be expressed with the following form;

$$Y = a + bx$$

- The function Y=a+bx is a straight line with two constant a and b, b is the coefficient of x, If x=0, then Y=a. the line cuts the Y-axis at a point 'a' called the intercept.
- Here we will fit a linear function;

$$Y = a + bx$$

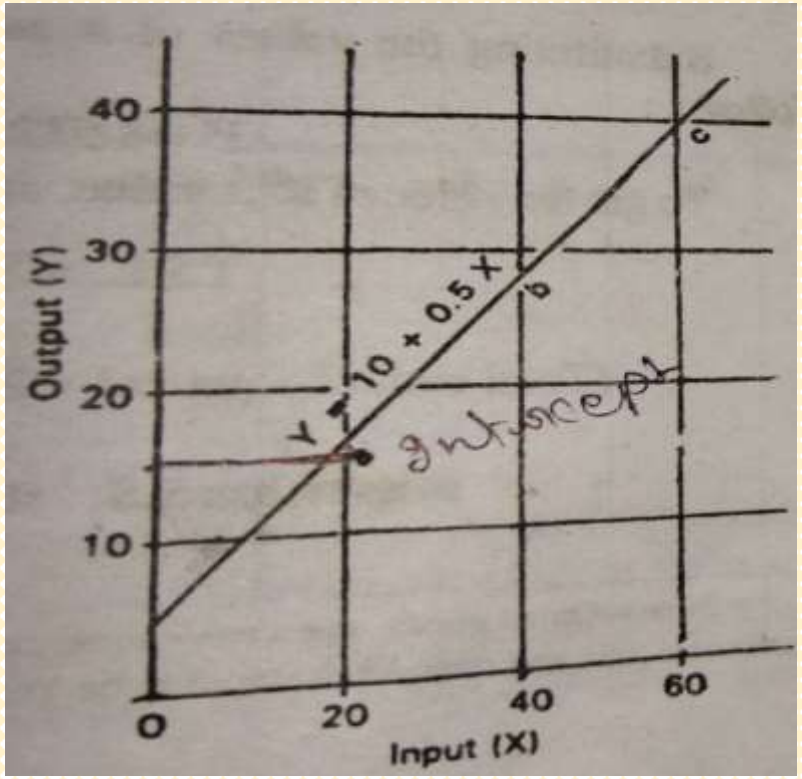
Where,

Y= dependent variable

X=Independent variable

A= constant

B=regression coefficient of Y on X



- The normal equation are;

$$\sum Y = Na + b\sum x$$

$$\sum XY = a\sum x + b\sum x^2$$

- To get the value of R^2, t-value and standard error of the coefficient.,

$$TSS = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

$$\text{RSS} = b\left[\sum XY - \frac{(\sum X)(\sum Y)}{N}\right]$$

$$R^2 = \frac{RSS}{TSS}$$

❑**Properties of functions;**

➢MP=AP

➢MP=constant

➢The MP and AP curves are horizontal straight lines that tend to coincidence.

Quadratic Production Function

- The word quadratic is equivalent to the Latin word 'Quartus' meaning 'Squared'.
- Since the square of any number is always positive, Y will take a positive value
- It is a second-degree function which is a simple form expressed as;

$$Y = aX^2$$

Where,

Y=output a= any number X^2 =input

or

$$Y = a + bX + cX^2$$

- In determine the value of a, b, and c the following normal equation are constructed;
- $\sum Y = na + b\sum(X) + c\sum(x^2)$
- $\sum XY = a\sum(x) + b\sum x^2 + c\sum(x^3)$
- $\sum X^2Y = a\sum(x^2) + b\sum(x^3) + c\sum(x^4)$
- Test of significance
- $TSS = \sum Y^2 - \frac{(\sum Y)^2}{N} = \sum(Y - \overline{Y})^2$
- $\sum(X - \overline{X})^2 = \sum X^2 - \frac{(\sum X)^2}{N}$
- $\sum(X - \overline{X})(Y - \overline{Y}) = \frac{\sum XY - (\sum X)(\sum Y)}{N}$
- S.S. (Linear regression)= $\sum(Y - \overline{Y})^2 - \frac{\{\sum(X - \overline{X})(Y - \overline{Y})\}}{\sum(X - \overline{X})^2}$
- S.S. (Quadratic regression)=a $\sum y + b\sum xy + c\sum x^2y - \frac{(\sum Y)^2}{N}$
- Deviation from linear regression= Total SS-SS(Linear regression)
- Deviation from quadratic regression=Total SS-SS (Quadratic regression)

Cobb-Douglas Production function

- The Cobb-Douglas Production Function is an example of two variable inputs power functions.
- The Cobb-Douglas production function was developed by Charles Cobb and Paul Douglas in 1928.
- The Cobb-Douglas production function is the most commonly used production function in agriculture.
- In a mathematical form, it is expressed as;
- Case of One independent variable

$$Y = aX^n$$

Taken both side logs;

$$\text{Log } Y = \text{Log } a + n \log x$$

Where,

Y=output,

X=Input

a=efficiency parameter,

n=elasticity of production with respect to input.

❖In determining the value of a, and x the normal equation;

- $\sum \log Y = N log\ a + b \sum logX$
- $\sum (laog\ X)(\log Y) = Log\ a \sum \log X + b \sum (lag\ X)^2$

❖Find out R^2, t-value, and standard error of the coefficient-

- $TSS = \sum (log\ Y)^2 - \frac{\sum (log\ Y)^2}{N}$
- $RSS = b\left[\sum (\log X) \log Y - \frac{\sum (\log X) \sum \log Y}{N}\right]$
- $R^2 = \frac{RSS}{TSS}$
- $t = \sqrt{F}$
- $SE\ of\ b = \frac{Regression\ coefficient}{t-value}$

❖Outcomes of Cobb-Douglas Production Function ;

- If $\alpha + \beta = 1$, represents a constant return to scale.
- If $\alpha + \beta < 1$, represents decreasing return to scale.
- If $\alpha + \beta > 1$, represents an increasing return to scale.

❖The Cobb-Douglas Production Function is homogeneous of degree one, which means if all the inputs are doubled, the output is doubled.

❖All inputs must be used for the output to be produced.

❖The managerial productivity of labour is proportion to the amount of production per unit of labour.

❖The managerial productivity of capital is proportion to the amount of production per unit of capital.

❖It allows either constant, increasing, or decreasing marginal productivity.

❖Exponents or b coefficient is the elasticity of production

❖If b=1 the marginal product (also Average product)

❖It was homogeneous of degree 1 with respect to the input bundle.

- ✓The elasticity of substitution between labour and capital in Cobb-Douglas production function is equal to unity because of this unit elasticity of substitution between two factors in production function iso-qant are convex to the origin.
- ✓Cobb-Douglas Production function can be extended by including more than two factors.
- ✓The return to scale is measured by the sum of exponents of Cobb-Douglas production function.
- ✓The shape of the iso-quant for Cobb-Douglas Production function is similar to the rectangular the hyperbola and position of iso-quant relative to the input axes depends on the magnitude of $\alpha \; and \; \beta$.
- ✓The elasticity of substitution Cobb-Douglas Production function is equal to one.
- ✓The expansion path of the Cobb-Douglas Production function has constant slop.
- ✓The expansion path of the Cobb-Douglas Production function is always a straight line.
- ✓The Marginal productivity of capital/labour is proportional to the amount of production per unit of capital labour.
- ✓Cobb-Douglas Production function is the constant return to scale.

Marginal Value Productivity (MVP)

- The marginal value product of inputs was estimated by following Formula;

$$MVP(X_j) = \frac{b_j \bar{Y}}{\bar{X}_j}$$

- Where,
- MVP= Marginal value of product of j^{th} input
- b_j = Production elasticity with respect to X_j
- $\bar{Y}$ = Geometric mean of the dependent variable Y
- $\bar{X}_j$ = Geometric mean of the independent variable X

THANK YOU