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Factor-Factor Relationship

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Factor-Factor Relationship

- This relationship deals with the resource combination and resource substitution.
- Cost minimization is the goal of factor -factor relationship.
- For the cost minimization at a given level of output we are required to compute the two ratio:

1) Marginal Rate of Substitution of Input
2) Price Ratio of Input

- Under factor-factor relationship, output is kept constant, input is varied in quantity.
- This relationship guides the producer in deciding 'How to produce'.
- This relationship is explained by the principle of factor substitution or principle of substitution between inputs.
- Factor -Factor relationship is concerned with the determination of least cost combination of resources.
- The choice indicators are substitution ratio and price ratio.

❖Factor-Factor relationship is also known as;

✓Substitution between Input

✓Rate of Technical Substitution

✓Principal of least cost combination

✓Production function with two variable input

❖Factor-factor law is represented in algebraic form as;

$$Y=F(X_1, X_2)$$

Where, Y is fixed level of output and X_1, X_2 are the quantities of variable inputs.

❖The factor-factor relationship deals with two independent variable giving rise to three dimensional diagram.

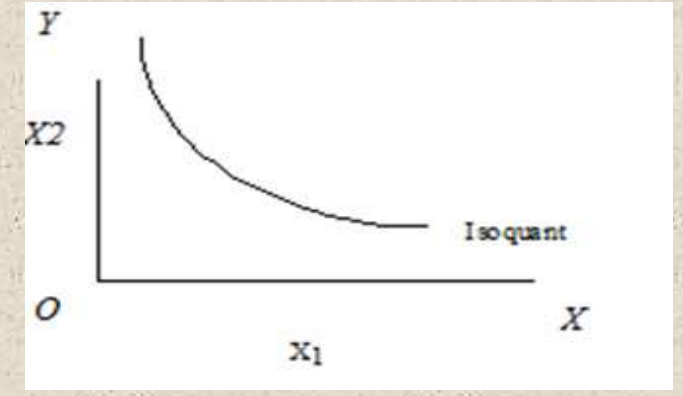
❖Iso-quant is a convenient method of compressing three—dimensional diagram into two-dimensional diagram.

Iso-quant

- Iso mean equal and quant mean quantity.
- Iso-quant is also termed as iso-product curve or equal product curve or product indifference curve.
- Iso-product curve show all the possible combination of two variable inputs, physically capable of producing the same amount of output or a given output level.
- Iso-quant curve similar to Indifference curve of theory of consumer behavior.

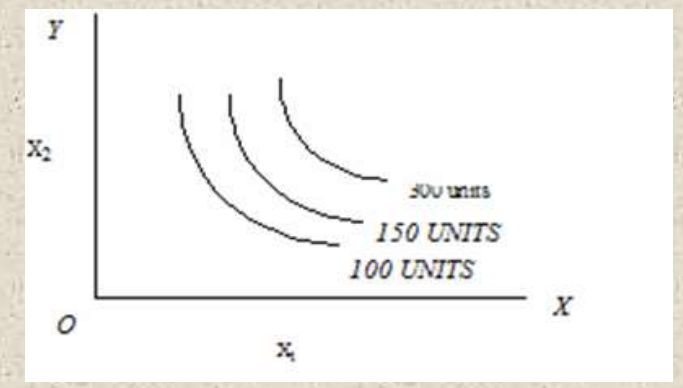
Table- Input Combination producing the same level of output

Input Combination	X1	X2
A	1	12
B	2	8
C	3	5
D	4	3
E	5	2



Iso-quant Map or Iso-product Contour

- If the number of isoquants are drawn on one graph, it is known as isoquant map.
- Isoquant map indicates the shape of production surface which in turn indicates the output response to the inputs.



Characteristics/Properties of Iso-quant

- They slop downward to the right, because if more of one input is used, less of another input will be employed at the given level of output.
- They are to the origin because of the diminishing marginal rate of substitution of one input for another.
- Iso-quant do not intersected. It is impossible to have tow different displayed from a single combination of input.
- Higher the output, the further away it is from the origin,.
- The slope of iso-quant denotes the rate of substitution between two resources.

Marginal Rate of Technical Substitution (MRTS)

- This is also known as the principal of diminishing MRTS.
- It is the rate of exchange between two productive resources, which are equally preferred.
- It refers to the amount by which one resource is reduced as another resource is increased by one unit.
- The quantity of one input to be sacrificed or given up in order to gain another input by one unit, in the process of substitution.
- MRTS of X_1 for X_2 is written as;

$$MRTS_{X_2X_1} = \frac{\Delta X_1}{\Delta X_2}$$

Where,

X_1=Added input

X_2=Replaced input

- MRTS is computed with the following equation;

$$\text{MRTS} = \frac{\text{Quantity of input sacrificed}}{\text{Quantity of input gained}}$$

Elasticity of Technical Substitution:

It is defined as the relative change in the factor proportion as a consequence of the relative change in MRTS.

$$\sigma = \frac{\text{proportion change in the ratio of factor used}}{\text{proportion change in MRTS}} = \frac{\frac{\Delta(K/L)}{K/L}}{\Delta(MRTS_{LK}/MRTS_{LK})}$$

Where,

L= labour and K= Capital

Substitutes

- A range of input combinations which will produce a given level of output.
- When one factor is reduced in quantity, a second factor must always be increased.
- MRTS is always less than zero.

Perfect Substitutes

- When two resources are completely interchangeable, they are called perfect substitutes.
- The isoquants for perfect substitutes is negatively sloped straight lines.
- The MRTS is constant.
- Example: Family labour and hired labour, Farm produced and purchased seed etc.,

Complements

- Two resources which are used together are called complements.
- In the case of complements reduction in one factor can not be replaced by an increase in another factor.
- MRTS is zero .

Perfect Complements

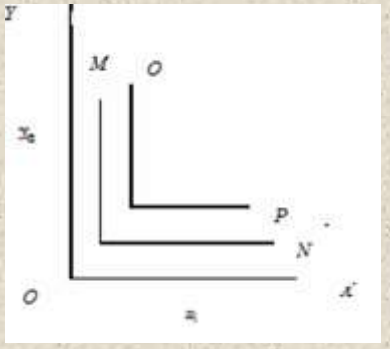
- Two resources which are used together in fixed proportion are called perfect complements.
- It means that only one exact combination of inputs will produce a particular level of output.
- The isoquant in this case is of a right angle.
- Ex: Tractor and driver, Pair of bullocks and labourer.

Types of Factor Substitution

The shape of isoquant and production surface will depend up on the manner in which the variable inputs are combined to produce a particular level of output. There can be three such categories of input combinations. They are:

1. ***Fixed Proportion combination of inputs***

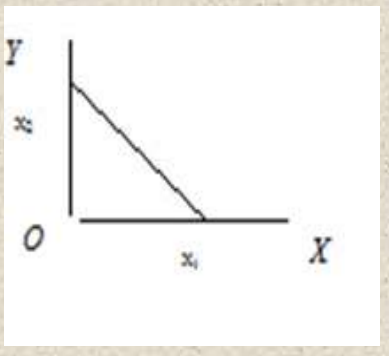
- To produce a given level of output, inputs are combined together in fixed proportion.
- Isoquants are 'L' shaped.
- It is difficult to find examples of inputs which combine only in fixed proportions in agriculture.
- An approximation to this situation is provided by tractor and driver combination. To operate another tractor, normally we need another driver.



2. Constant rate of Substitution:

- For each one unit gain in one factor, a constant quantity of another factor must be sacrificed.
- When factors substitute at constant rate, isoquants are linear, negatively sloped.
- When inputs substitute at constant rate, it is economical to use only one resource, and which one to use depends up on relative prices.
- Algebraically, constant rate of factor substitution is expressed as;

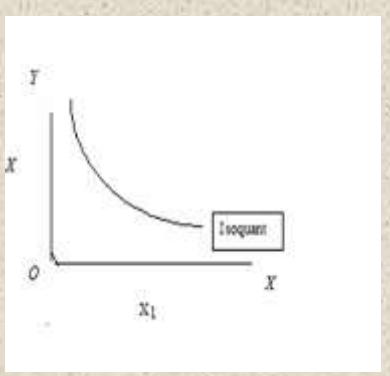
$$\Delta_1X_2/\ \Delta_1X_1 = \Delta_2X_2/\ \Delta_2X_1 = \ldots\ldots = \Delta_nX_2/\ \Delta_nX_1$$



3.Decreasing Rate of substitution:

- Each one unit increase in one factor requires smaller and smaller sacrifice in another factor.
- Example; Capital & labour, concentrate &fodder, organic &inorganic fertilizer etc.
- Isoquants are convex to the origin when inputs substitute at decreasing rate.
- Algebraically, decreasing rate of substitution is expressed as

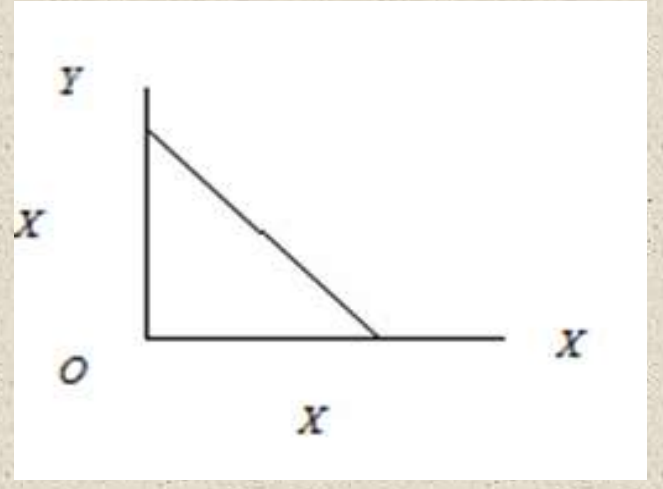
$$\Delta_1X_2/\ \Delta_1X_1 > \Delta_2X_2/\ \Delta_2X_1 > \ldots\ldots > \Delta_nX_2/\ \Delta_nX_1$$



Iso-Cost line

- Iso-cost Line is also termed as price line, budget line, iso outlay line, factor cost line
- Iso-cost line defines all possible combinations of two resources (X_1 and X_2) which can be purchased with a given outlay of funds.
- Characteristics of Iso-cost line;

- ✓ As the total outlay increases, the iso-cost line moves farther away from the origin.
- ✓ Iso-cost line is a straight line because input prices do not change with the quantity purchased.
- ✓ The slope of iso-cost line indicates the ratio of factor prices.



Least-Cost Combination

- There are innumerable possible combinations of factors which can be used to produce a particular level of output.
- The problem is to find out a combination of inputs which should cost the least, a cost minimization problem.
- There are three methods to find out the least cost combination of inputs. They are:

1. ***Tabular Method/Simple Arithmetical calculations:***

   - One possible way to determine the least cost combination is to compute the cost of all possible combinations of inputs and then select one combination with minimum cost.
   - This method is suitable where only a few combinations produce a particular level of output.

X1	X2	X1@Rs.3	X2@Rs.2	Total cost
10	3	30	6	36
7	4	21	8	29
5	6	15	12	27
3	8	9	16	25
2	12	6	24	30

The above table shows five combinations of inputs which can produce a given level of output. The price per unit of X1 is Rs.3/ - and of X2 is Rs.2/-. The total cost of each combination of inputs is computed. Out of five combination, 3 units of X1 and 8 units of X2 is the least cost combination of inputs i.e., Rs.25/-

2. Algebraic Method

Step-1: Compute the marginal rate of technical substitution

$$MRTS = \frac{\text{Number of units of replaced resource}}{\text{Number of units of added resource}}$$

$$MRTS_{X_1X_2} = \frac{\Delta X_2}{\Delta X_1} \text{ (When we substitute } X_1 for\ X_2)$$

$$MRTS_{X_2X_1} = \frac{\Delta X_1}{\Delta X_2} \text{ (When we substitute } X_2 for\ X_1)$$

Step-2: Compute the inverse price ratio (PR)

$$PR = \frac{\text{Price per unit of added resource}}{\text{Number of units of added resource}\text{Price per unit of replaced resource}}$$

$$MRTS_{X_1X_2} = \frac{Px_1}{Px_2} \text{ (When we substitute } X_1 for\ X_2)$$

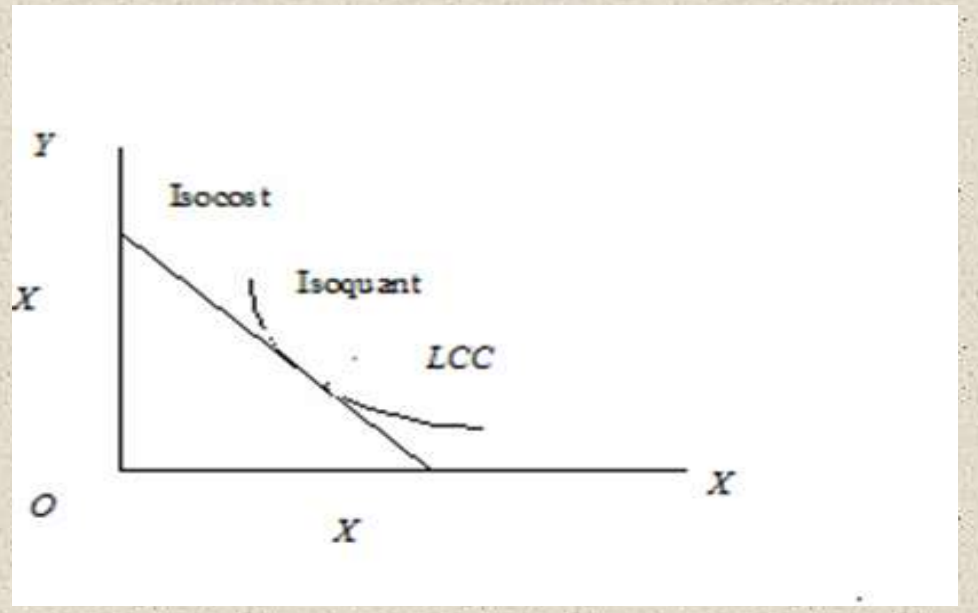
$$MRTS_{X_2X_1} = \frac{Px_2}{Px_1} \text{ (When we substitute } X_2 for\ X_1)$$

Step-3: Finding the least cost combination by equating marginal rate of technical substitution with inverse price ratio;

$$\frac{\Delta X_2}{\Delta X_1} = \frac{Px_1}{Px_2} \text{ or } \qquad \frac{\Delta X_1}{\Delta X_2} = \frac{Px_2}{Px_1}$$

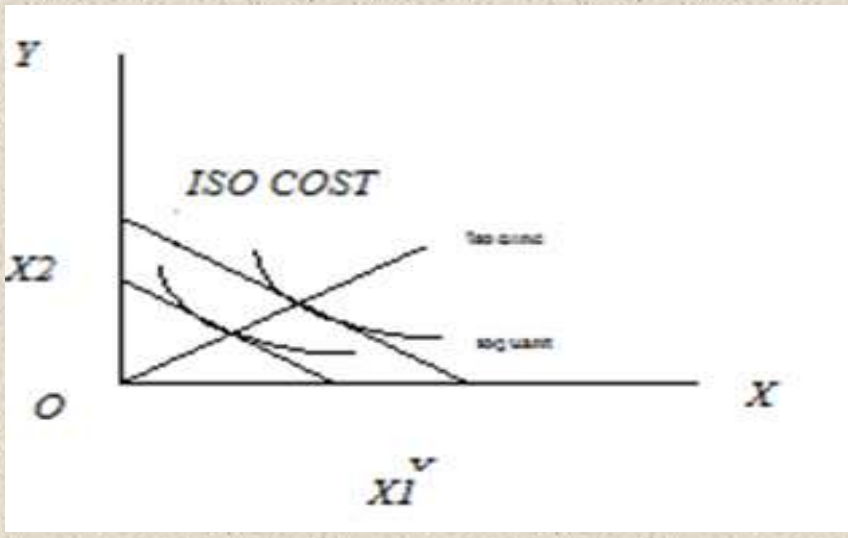
3. Graphical Method

- The slope of isoquant indicates MRTS and the slope of iso-cost line indicates factor price ratio, minimum cost for given output will be indicated by the tangency of these isoclines.
- For this purpose, iso-cost line and isoquant are drawn on the same graph for different levels of production.
- The least cost combination will be at the point where iso-cost line is tangent to the isoquant i.e., slope of isoquant=slope of iso-cost line i.e., MRS=PR.



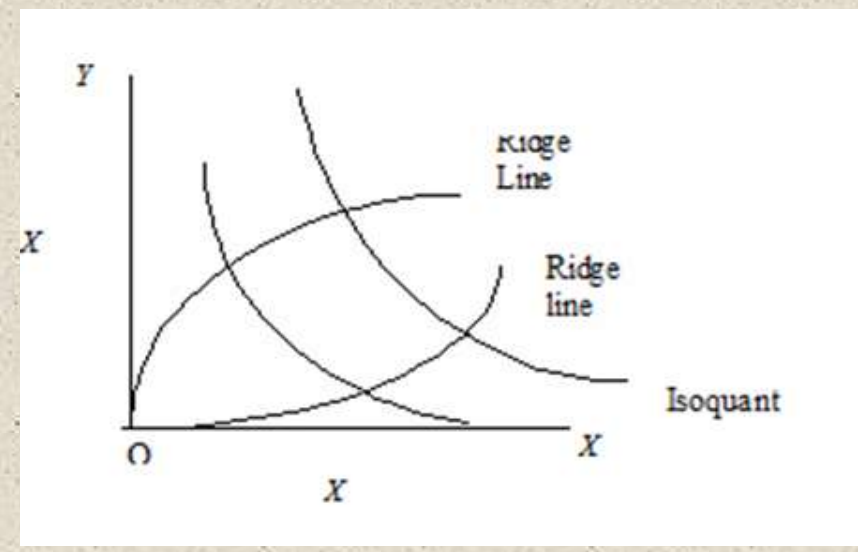
Iso-clines/Expansion path

- There can be number of possible output levels as shown in the figure and the least cost combination can be found out for these various output levels.
- A line or curve connecting the least cost combination of inputs for all output levels is called isocline.
- The isocline passes through all the isoquants at points where they have the same slope.
- Isoclines can be drawn at different sets of price ratio.
- All isoclines of course converge at the point of maximum output.
- Though all the points on isocline represent least cost combination, only one point represents the maximum profit output.



Ridge Line

- Ridge lines represent the points of maximum output from each input, given a fixed amount of another input.
- On the ridge lines MPP is zero.
- Ridge lines represents the economic relevance, the boundaries beyond which isoquant map ceases to have economic meaning.
- The portions of isoquants which lie between the lines are suited for economic production (Where MPP of both inputs are positive but decreasing and isoquants are negatively sloped).
- Portions of iso-quants outside the ridge lines are not suitable for production in economic terms (outside the ridge lines, MPP of both factors are negative and methods of production are inefficient).



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